1 $u_x . u_y = 1.$ $\qquad (x_0, y_0, u_0) = (2s, 0, 5s).$

$$\partial x/\partial t = q \qquad 2p_0 = 5 \Rightarrow p_0 = 2{,}5$$
$$\partial y/\partial t = p. \qquad p_0 . q_0 = 1. \Rightarrow q_0 = 0{,}4.$$
$$\partial u/\partial t = 2pq$$
$$\left.\begin{array}{l}\partial p/\partial t = 0 \\ \partial q/\partial t = 0.\end{array}\right\} \begin{array}{l} p = 2{,}5 \\ q = 0{,}4. \end{array}$$

$$x = 0{,}4t + 2s. \Rightarrow s = \tfrac{1}{2}(x - 0{,}16y).$$
$$y = 2{,}5t \Rightarrow t = 0{,}4y.$$
$$u = 2t + 5s.$$
$$\Rightarrow u = 0{,}8y + 2{,}5x - 0{,}4y. = 0{,}4y + 2{,}5x.$$

$$u_x . u_y = 0{,}4 . 2{,}5 = 1.$$
$$u(2x, 0) = 0{,}4.0 + 2{,}5(2x) = 5x \qquad \boxed{u = 0{,}4y + 2{,}5x}$$

2. $$\left.\begin{array}{l} a = (1+x^2)(4+x^2). \\ b = \tfrac{1}{2}(5+2x^2) \\ c = 1 \end{array}\right\} b^2 - ac = \tfrac{1}{4}(5+2x^2)^2 - (1+x^2)(4+x^2) = \frac{25}{4} - \frac{16}{4} = \underline{\underline{(3/2)^2 > 0.}}$$

$b^2 - ac > 0 \Rightarrow$ hyperbolisch.

ik ben op zoek naar oplossingen van.

$$\psi_x - \frac{-b \pm \sqrt{b^2 - ac}}{a.} \psi_y = 0.$$
$$\psi_x + \frac{(5+2x^2) \mp 3.}{2(1+x^2)(4+x^2)} \psi_y = 0.$$

1) $\psi_x + \frac{1}{(4+x^2)} \psi_y = 0$ $\qquad$ of 2). $\theta_x + \frac{1}{(1+x^2)} \theta_y = 0$

1). $\frac{\partial y}{\partial x} = \frac{-1}{(4+x^2)}.$ $\qquad \frac{\partial y}{\partial x} = -\frac{1}{(1+x^2)}.$

$$y = -\int \frac{1}{(4+x)^2} dx. \qquad \boxed{y = -\arctan x + C.}$$

$$y = -\int \frac{1}{4+x^2}\,dx \qquad x/2 = p. \qquad dx = 2.dp.$$

$$= -\frac{1}{4}\int \frac{1}{1+(x/2)^2}\,dx$$

$$= -\frac{1}{2}\cdot\int \frac{1}{1+p^2}\,dp =$$

$$= \boxed{-\frac{1}{2}\,\text{arctang}\,\frac{x}{2} + C. = y}$$

Ik reken even verder omdat mij onduidelijk is wat met karakteristieken wordt bedoeld.

kies. $(x_0, y_0) = (0,0)$.

$$\varphi_x + \frac{1}{(4+x^2)}\varphi_y = 0. \qquad (x(0) = x_0 + \; y(0) = y_0 + S \qquad \phi_0 = S.$$

$$\partial x/\partial t = 1 \Rightarrow x = t$$
$$\partial y/\partial t = \frac{1}{4+t^2} \Rightarrow y = \frac{1}{2}\,\text{arctang}\,\cancel{t}/2 + S.$$

$$\partial\phi/\partial t = 0 \Rightarrow \phi = S$$

$$\boxed{\phi = y - \tfrac{1}{2}\,\text{arctang}\,x/2.}$$

Helfde voor Θ wordt.

$$\boxed{\Theta = y - \text{arctang}\,x}$$

Woonplaats: | Jaar van eerste inschrijving: | Naam docent:

3a ongedempt. $u_{tt} = c u_{xx} \Rightarrow T''/cT = \dfrac{X''}{X} = -\lambda^2$

$u(0,t) = u(\pi,t) = 0 \Rightarrow X(0) = X(\pi) = 0$

$\Rightarrow X = A_n \sin nx \Rightarrow n = \lambda = k \in \mathbb{N}$

gedempt. $u_{tt} = c u_{xx} - 2\alpha u_t \Rightarrow \dfrac{T''}{cT} = \dfrac{X''}{X} - \dfrac{2\alpha}{c}\dfrac{T'}{T}$

$T''/cT + \dfrac{2\alpha}{c^2}\dfrac{T'}{T} = \dfrac{X''}{X} = -\lambda^2$

nu geldt dus ~~voor~~ $X'' + \lambda^2 X = 0 \Rightarrow X = A \sin nx$

$n = \lambda = k \in \mathbb{N}$

b $T'' + 2\alpha T' + c^2 n^2 T = 0.$

probeer $T = Be^{\lambda t}$

$\Rightarrow Be^{\lambda t}(\lambda^2 + 2\alpha\lambda + c^2n^2) = 0.$

$\lambda_\pm = -\alpha \pm \sqrt{\alpha^2 - c^2n^2}$

$n = k.$ $u_t(x,0) = 0 \Rightarrow T'(0) = 0.$

i Stel. $\alpha^2 - c^2n^2 > 0 \Rightarrow T(t) = Me^{\lambda_+ t} + Ne^{\lambda_- t}$

$T'(t) = \lambda_+ Me^{\lambda_+ t} + \lambda_- Ne^{\lambda_- t}$

$\boxed{\lambda_+ M = -\lambda_- N}$

ii Stel $\alpha^2 - c^2n^2 = 0 \Rightarrow T(t) = (M + Nt)e^{\lambda t}$

$T'(t) = e^{\lambda t}(\lambda M + \lambda N t + N).$

$\Rightarrow \boxed{\lambda M = -N}$

iii Stel $a^2 - c^2n^2 < 0 \Rightarrow T(t) = e^{-\alpha t}\left[A\cos\sqrt{\alpha^2 - c^2n^2}\,t + B\sin\sqrt{\alpha^2 - c^2n^2}\,t\right]$

$T'(t) = e^{-\alpha t}\left[(-\alpha A + B\sqrt{c^2n^2 - \alpha^2})\cos pt + (-\alpha B - Ap)\sin pt\right]$

$p = \sqrt{c^2n^2 - \alpha^2}$

$\Rightarrow -\alpha A + Bp = 0.$

c. (i) en (iii) komen eigenlijk op hetzelfde neer. maar (ii) is wel degelijk anders.

blijkbaar komt (ii) niet voor in het algemeen.

$u(x,t) = X(x) \cdot T(t).$

$X(x) = A \sin kx$ (zie a).

$T(t) = A\exp\left[-a + i\sqrt{k^2c^2 - a^2}\right]t + B\exp\left[-a - i\sqrt{k^2c^2-a^2}\,t\right].$

Omschrijven naar geometrische functies $e^{x+yi} = e^{x}[\cos x + i\sin x]$ levert het gevraagde

d. $u(x,0) = f(x)$

$\Rightarrow \quad f(x) = \sum a_k \sin kx \Rightarrow \boxed{a_k = \frac{2}{\pi}\int_0^{\pi} f(x)\sin kx\,dx}$

$u_t(x,0) = 0.$

$= T'(t) \cdot X(x)$ zie ook opgave b.

$u_t(x,t) = \sum \sin kx\left[(-\alpha a_k + b_k p)\cos pt + (-\alpha b_k - a_k p)\sin pt\right]e^{-\alpha t}$

$u_t(x,0) = \sum \sin kx\left[-\alpha a_k + b_k p\right] = 0.$

wegens orthoganaliteit moeten alle coeficienten. $\mu_n = 0.$

$\Rightarrow \boxed{b_k = \frac{\alpha}{p}a_k}$ met $p = \sqrt{k^2c^2 - \alpha^2}$

Woonplaats: | Jaar van eerste inschrijving: | Naam docent:

4a. $u_t = u_{xx} + x(\pi - x)$.

$u_t = V_t = u_{xx} + x(\pi - x) = V_{xx} + \overbrace{w'' + x(\pi - x)}^{=0}$

$w'' = -x(\pi - x)$

$\Rightarrow \quad w = \frac{1}{12}x^4 - \frac{\pi}{6}x^3 + Ax + B.$

$u(x,0) = V(x,0) + w(x)$

$u(0,t) = V(0,t) + w(0) = 0$ (kies $w(0) = 0$). $\Rightarrow B = 0$.

$u(\pi,t) = V(\pi,t) + w(\pi) = 0$ (kies $w(\pi) = 0$) $\Rightarrow$

$A = \pi^3\left(\frac{1}{6} - \frac{1}{12}\right) = \frac{\pi^3}{12}.$

Nu nog op te lossen
$$\boxed{\begin{aligned} &V_t = V_{xx} \\ &V(x,0) = f(x) - w(x) \\ &V(0,t) = V(\pi,t) = 0 \end{aligned}}$$

met $w(x) = \frac{1}{12}\left(x^4 - 2\pi x^3 + \pi^3 x\right)$.

b. Ik vraag me af waarom ik $w(x)$ in vraag a al heb bepaald???

oplossen van $V = (X(x)T(t))$

$\Rightarrow$

$$V = \sum C_n . e^{-n^2 t} \sin nx.$$

$u(x,t) = \sum C_n e^{-n^2 t} \sin nx + w(x)$.

$\lim_{t \to \infty} u(x,t) = w(x)$ hmmm..... dat wordt idd ook gesuggereerd in de vraag.

Stel $u(x,t) = \sum C_n(t) e^{-n^2 t} \sin nx$

en $x(\pi - x) = \sum F_n \sin nx . e^{-n^2 t}$

$$\sum e^{-n^2 t} \sin nx \left[-n^2 C_n + F_n + n^2 C_n t - C_n'\right].$$

$F_n = C_n'$

Zie pag 4 voor verdere uitleg

5 $$\int_\Omega \left((\nabla w)^2 + w \Delta w \right) d\Omega = \int_{\partial\Omega} w \frac{\partial w}{\partial n} \, dS$$

Stel er zijn twee oplossingen: w_1, w_2.

dus $w(x) \geqslant 0$ en $w_2(x) \geqslant 0$ $x \in \Omega$.

en $\Delta w_1 = w_1^2$ en $\Delta w_2 = w_2^2$ op Ω

en $w_1 = f$ en $w_2 = f$ op $S. = \delta\Omega$

$$\int_{\partial\Omega} (\nabla w_1)^2 + w_1^3 \, d\Omega = \int_{\partial\Omega} f \frac{\partial t}{\partial n} \, dS = Q$$

$$\int (\nabla w_2)^2 + w_2^3 \, d\Omega = Q.$$

Bekijk de verschil functie $g = w_1 - w_2$.

$$\Delta g = \Delta w_1 - \Delta w_2 = w_1^2 - w_2^2 = g \cdot (w_1 + w_2).$$

$g = 0$ op de rand.

Stel $\exists x$ met $g(x) > 0$ $(x \in \Omega)$

dan heeft g een maximum in $\Omega - \delta\Omega$.

(noem dat maximum p).

g continue $\Rightarrow$ ~~er is~~ $\exists$omgeving $P \subset \Omega$ met $p \in P$ en

$g(x) \geqslant 0$. $\forall x \in P$.

$\Rightarrow \Delta g \geqslant 0$ in P.

Pas nu Maximum minimum toe

$$\Rightarrow g(p) \leqslant \max_{x \in \delta(p)} g(x)$$

Nu moet dus gelden $g(p) = g(x)$ $\forall x \in \delta(p)$.

$\Rightarrow$ Dit voortzetten met andere P's levert $g = \text{constant} > 0$.

maar dit is in strijd met $g = 0$ op de rand.

contradictie.

Gebruik hetzelfde om aan te tonen dat $-g \leqslant 0$.

$$\Rightarrow g = 0 \quad \Rightarrow w_1 = w_2.$$

Vreemd... ik heb groen niet gebruikt.

Woonplaats: | Jaar van eerste inschrijving: | Naam docent:

5 Stel $\exists$ 2 functies. ω_1 en ω_2 die voldoen aan het probleem.

Bekijk $g = \omega_1 - \omega_2$. $\quad g = 0$ op de rand.

$$\Delta g = g \cdot (\omega_1 + \omega_2).$$

$$\int_\Omega (\nabla g)^2 + g^2 (\omega_1 + \omega_2) \, d\Omega = 0.$$

$(\nabla g)^2 \geqslant 0.$ ~~(axioma van het inproduct~~

$\Rightarrow$ óf $g = 0 \quad \omega_1 = \omega_2$

óf $\omega_1 + \omega_2 = 0. \Rightarrow \omega_1 = -\omega_2.$

$\Rightarrow \omega_1 = \omega_2 = 0$ (ω_1 en ω_2 waren positief)

maar dit voldoet niet aan $\omega_1 = \omega_2 = f$ op de rand.

Vervolg 4b.

$$F_n e^{-n^2 t} = \frac{2}{\pi} \int_0^\pi (\pi x - x^2) \sin nx \, dx.$$

$$-\frac{2}{\pi n} (\pi x - x^2) \cos nx \Big|_0^\pi + \frac{2}{n\pi} \int_0^\pi \left(\pi - \tfrac{1}{2} x\right) \cancel{\sin} \cos nx \, dx$$

$$= \Big|_0^\pi + \frac{\cancel{4}}{n^2 \pi} \int_0^\pi \sin nx \, dx. = \frac{-1}{n^3 \pi} \left[(-1)^n - 1\right].$$

$\cancel{C_n(t) - C_n(0)}$ $C_n(t) - C_n(0) = \int_0^t F_n(\tau) \, d\tau$

$$= \frac{1}{n^3 \pi} \left[1 - (-1)^n\right] \int_0^t e^{n^2 \tau} \, d\tau$$

$u(x,0) = f(x) = \sum C_n(0) \sin nx$

$$\Rightarrow C_n(0) = \frac{2}{\pi} \int_0^\pi f(x) \sin nx.$$

$$\lim_{t\to\infty} u(x,t) = \lim \sum \left[\frac{\frac{2}{\pi}\int f(x)\sin nx + \frac{1}{n^3\pi}[1-(-1)^n]\int_0^t e^{n^2\tau}\,d\tau}{e^{n^2 t}} \right] \sin nx .$$

$$= \sum \cancel{\lim}\ \frac{1}{n^3\pi}[1-(-1)^n] \lim \frac{\int_0^t e^{n^2 t}\,dt}{e^{n^2 t}} \sin nx .$$

(L'Hopital).

$$= \sum \frac{1}{n^3\pi}[1-(-1)^n] \frac{e^{n^2 t}}{n^2 e^{n^2 t}} \quad \sin nx .$$

$$= \sum \frac{1}{n^5\pi}[1-(-1)^n] \sin nx .$$

wellicht is dit de fourier reeks van $w(x) = \frac{1}{12}(x^4 - 2\pi x^3 + \pi^3 x)$

$$W_n = \frac{2}{\pi}\int_0^\pi \frac{1}{12}(x^4 - 2\pi x^3 + \pi^3 x) \sin nx \, dx .$$

$$= \quad + \frac{1}{6\pi n}\int (4x^3 - 6\pi x^2 + \pi^3) \cos nx \, dx .$$

$$= \frac{-12}{6\pi n^2}\int (x^2 - \pi x) \sin nx . \quad \text{(deze is al uitgerekend.)}$$

$$= \frac{1}{n^2}\left[\frac{1}{n^3\pi}(1-(-1)^n\right] . \qquad \text{warempel !!!}$$

$$\lim_{t\to\infty} u(x,t) = -\sum W_n \sin nx . = -w(x)$$

~~logisch want~~

~~$w(x,t) =$~~